SELF-OSCILLATIONS DURING THE WEDGING OF THIN BODIES

(OB AVTOKOLEBANIIAKH PRI RASKLINIVANII TONKIKH TEL)

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The present paper forms a brief supplement to [1], which gave a theoretical description of self-oscillatory phenomena occurring during the wedging of an infinite brittle body by a rigid wedge moving at constant velocity. Starting from the assumption that the cohesion modulus depends on the velocity of the tip of the crack, initially decreasing with increase in this velocity, it was shown that the regime of crack propagation at constant velocity is unstable if the wedging velocity is low, and that the crack develops in an oscillatory manner.

In experimental work it would be impractical to use plates which are large enough to satisfy the conditions of the theory of wedging of an infinite body. Conversely, it is not possible to carry out a satisfactory analytical investigation for arbitrary finite plates. However, there exist two limiting cases which admit a very simple analytical study and which, in addition, can be reproduced sufficiently closely in the laboratory: the wedging of a thin beam and the planing of a thin shaving off a large body. The corresponding static problems of wedging have been solved by Obreimov [2] and by Roesler and Benbow [3]. Here we shall investigate self-oscillations during wedging for these two extreme cases. It is considered that a study of self-oscillations during the planing of a thin shaving can throw more light on the nature of the self-oscillations which occur during cutting.

1. Let us consider the following problems.

1) A thin beam of depth $2H$ and width b split in two by a rigid wedge of depth $2h$ moving at a constant velocity V (Fig. 1).

2) A thin shaving of thickness H planed from a very large body of

width b by a rigid wedge of depth h moving at a constant velocity V (Fig. 2).

As in [1], we make the assumption that the density of surface energy T and the cohesion modulus K depend on the instantaneous velocity v of the tip of the crack, and that with increase in v from zero to $v = v_*$ both T and K decrease and then start to increase.

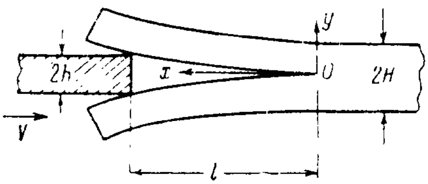

Fig. 1.

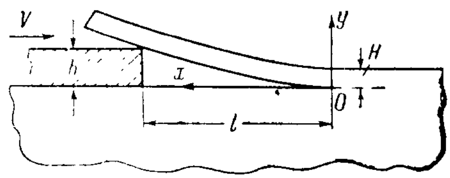

Fig. 2.

During the wedging process the length l of the crack varies, so that the velocity of its tip is given by $v = V + dl/dt$.

In both cases, we assume that the wedged bodies deform in accordance with the beam theory and that the beam is rigidly clamped at the tip of the crack. To this approximation the force applied by the wedge and the cohesive forces acting at the tip of the crack deform only that part of the material lying between them. Therefore, the work done by these forces is equal to the change of energy of this part only. Consequently, the equation of energy balance in both cases is of the form

$$\frac{d\varepsilon}{dt} + \frac{d\Pi}{dt} = FV - 2T(v)\, vb \tag{1.1}$$

Here ε and Π are the kinetic and potential energies of the material situated between the leading edge of the wedge and the tip of the crack, and F is the wedging force applied by the wedge to the body.

Consider an auxiliary motion in which the tip of the crack is stationary ($v = 0$) and the quantities l, $\dot{l}$, ... at a given instant coincide with the corresponding quantities in the basic motion. The equation of energy balance for the auxiliary motion is

$$\frac{d\varepsilon'}{dt} + \frac{d\Pi'}{dt} = -F' \frac{dl}{dt} \tag{1.2}$$

where ε', Π' and F' are the corresponding quantities for the auxiliary motion. Since the velocity of the wedge is small compared with the velocity of sound we can proceed as in [1] and take $F' = F$ and $d\Pi'/dt = d\Pi/dt$, so that after subtracting (1.2) from (1.1) we obtain the fundamental equation

$$\frac{d(\varepsilon-\varepsilon')}{dt} = [F - 2T(v)\,b]\,v \tag{1.3}$$

2. In order to evaluate the quantities ε, ε' and F we make use of the quasistatic approximation, which is possible in view of the low velocity of the wedge. For Problem 1 we have

$$\varepsilon = 2\cdot\frac{1}{2}\rho bH\int_0^l\left(\frac{dy}{dt}\right)^2 dx \tag{2.1}$$

where $y(x, t)$ is the deflection curve of the beam and ρ is the density.

Following the procedure of [3] we assume that at $x = 0$ the beam is rigidly clamped, so that the static deflection distribution is given by

$$y = 3h\left(\frac{1}{2}\xi^2 - \frac{1}{6}\xi^3\right), \qquad \xi = \frac{x}{l} \tag{2.2}$$

We have

$$\frac{dy}{dt} = \frac{\partial y}{\partial x}v + \frac{\partial y}{\partial l}\frac{dl}{dt} = \frac{3h}{l}(v-\xi)(v-\xi\dot{l})\left(\xi - \frac{1}{2}\xi^2\right) \tag{2.3}$$

Substituting (2.3) into (2.1) and performing the necessary simple operations, we find that

$$\frac{d(\varepsilon-\varepsilon')}{dt} = \frac{3}{4}\,\frac{\rho bHh^2}{l}\,v\ddot{l} \tag{2.4}$$

Here we have discarded terms containing first derivatives. This can be done since the inertia is small (see the corresponding estimates in [1]).

We see also that in the same quasistatic approximation

$$\Pi = \frac{3EIh^2}{l^3}, \qquad I = \frac{bH^3}{12}, \qquad F = -\frac{\partial\Pi}{\partial l} = \frac{3EbH^3h^2}{4l^4} \tag{2.5}$$

Here E is Young's modulus. Substituting (2.4) and (2.5) into (1.3) we obtain the fundamental differential equation for the function $l(t)$

$$\frac{d^2 l}{dt^2} = \frac{A}{l^3} - BK^2(v)\,l, \quad A = \frac{EH^2}{\rho}, \quad B = \frac{8(1-\nu^2)}{3\pi E\rho Hh^2} \tag{2.6}$$

where ν is Poisson's ratio. Completely analogously, we obtain the same equation (2.6) for Problem 2, but with the coefficient B given by

$$B = \frac{16(1-\nu^2)}{3\pi E\rho Hh^2} \tag{2.7}$$

3. In equation (2.6) it is convenient to transfer to non-dimensional form and set

$$\Lambda = \frac{l}{l_*(0)}, \qquad \tau = \frac{v_1 t}{l_*(0)}, \qquad f\left[\frac{V}{v_1} + \frac{d\Lambda}{d\tau}\right] = \frac{K^2(v)}{K^2(0)} \tag{3.1}$$

where v_1 is the characteristic velocity, which can be selected in various ways, $l_*(0)$ is the free crack length at zero velocity given by

$$l_*(0) = H\left(\frac{3\pi E^2 h^2}{8K^2(0)H(1-\nu^2)}\right)^{1/4} \quad \text{(Problem 1)}$$

$$l_*(0) = H\left(\frac{3\pi E^2 h^2}{16K^2(0)H(1-\nu^2)}\right)^{1/4} \quad \text{(Problem 2)} \tag{3.2}$$

Equation (2.6) now becomes

$$\alpha \frac{d^2\Lambda}{d\tau^2} = \frac{1}{\Lambda^3} - f\left(\frac{V}{v_1} + \frac{d\Lambda}{d\tau}\right)\Lambda \tag{3.3}$$

where

$$\alpha = \left(\frac{v_1}{c}\right)^2 \sqrt{\frac{3\pi}{1-\nu^2}} \frac{Eh}{2K(0)\sqrt{2H}} \quad \text{(Problem 1)}$$

$$\alpha = \left(\frac{v_1}{c}\right)^2 \sqrt{\frac{3\pi}{1-\nu^2}} \frac{Eh}{4K(0)\sqrt{H}} \quad \text{(Problem 2)} \qquad c = \sqrt{\frac{E}{\rho}} \tag{3.4}$$

Investigation of equation (3.3) shows that, as in [1], when $V > v_*$ steady wedging is stable with respect to small disturbances. When $V < v_*$ steady wedging is unstable and there exists a self-oscillatory regime of crack propagation. In general the self-oscillations can be of two types: with or without intervals when the tip of the crack is stationary. The wavelength of the oscillations increases with increase in the velocity of the wedge. In order to calculate the self-oscillations set up, it is necessary to specify the function f in some definite form and to perform the integration numerically. An investigation of limiting cases may be undertaken in an exactly analogous manner [1].

It is known that at low cutting velocities the shaving is found to be ribbed, the system of transverse ribs being approximately periodic; the ribs thin out as the cutting velocity is increased and disappear completely at a velocity above some critical value. It is possible that this phenomenon may be explained by the self-oscillation process considered here, combined with oscillations of the tip of the free crack formed in front of the cutter. In applying the approach developed here to the problem of cutting it must be borne in mind that, for thin shavings, the density of surface energy and the cohesion modulus may be found to depend on the thickness of the shaving, since the plastic region in the vicinity of the tip of the crack might extend right through the shaving.

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